

II. METHOD OF MEASUREMENT

A. The Differential Method

Rutherford¹² first introduced in 1911 the differential method, which is well suited for the accurate determination of a small difference $\Delta\lambda$ in the decay constants of two sources. $\Delta\lambda$ is determined from the variation with time of the differential current obtained when the intensities of the sources 1 and 2 are compared in two essentially identical ionization chambers *A* and *B* connected to collect ions of opposite sign.

The sensitivities of the chambers are defined as S_A and S_B , with a mean sensitivity $\bar{S} = (S_A + S_B)/2 \equiv 1$, so that the intensities of the sources at $t=0$ are $\bar{S}I_1$ and $\bar{S}I_2$ divisions of 10^{-14} ampere on the recorder. The difference currents are

$$i_{12} = S_A I_1 e^{-\lambda t} - S_B I_2 e^{-\lambda t} e^{-\Delta\lambda t}, \quad (1)$$

and

$$i_{21} = S_A I_2 e^{-\lambda t} e^{-\Delta\lambda t} - S_B I_1 e^{-\lambda t} \quad (2)$$

when the sources are interchanged. (The disintegration constant λ is associated with source 1 and $\lambda + \Delta\lambda$ with source 2.)

The average value of the difference currents at time t is

$$\Delta i = (i_{12} - i_{21})/2 = \bar{S}(I_1 - I_2)e^{-\lambda t} + \bar{S}I_2 e^{-\lambda t} \Delta\lambda t. \quad (3)$$

The difference in the chamber sensitivities is obtained from

$$\Delta i_s = (i_{12} + i_{21})/2 = (S_A - S_B)(I_1 + I_2)e^{-\lambda t}/2 - (S_A - S_B)I_2 e^{-\lambda t} \Delta\lambda t/2. \quad (4)$$

In individual experiments on technetium the sensitivities of the chambers S_A and S_B differed over a range of 0 to 10 parts in 10 000 for the chambers used.¹³

B. Calculation of $\Delta\lambda/\lambda$

Since Δi as given in Eq. (3) is not convenient for analysis, it is multiplied by $e^{\lambda t}$ following the practice of Segrè and Wiegand. The resulting equation in t (neglecting the adjustment for chamber sensitivities), $e^{\lambda t} \Delta i = I_1 - I_2 + I_2 \Delta\lambda t$, can be fitted by a least squares analysis to the linear form $y = a + bx$. Here $y = e^{\lambda t} \Delta i$, $x = t$, $a = I_1 - I_2 = i_0$, the initial difference in intensity of the two sources, and $b = I_2 \Delta\lambda$, where I_2 will henceforth be called I_0 , the initial current of the source with decay constant $\lambda + \Delta\lambda$. The calculations for a and b and their standard deviations σ_a and σ_b were made by a procedure outlined by Birge.¹⁴

¹² E. Rutherford, Wien. Ber. 120, 303 (1911).

¹³ S_{A1} for source 1 in chamber *A* may differ from S_{A2} for source 2 in chamber *A*, and strictly

$e^{\lambda t} \Delta i = (S_{A1} + S_{B1})I_1/2 - (S_{A2} + S_{B2})I_2/2 + (S_{A2} + S_{B2})I_2 \Delta\lambda t/2$.

No error is introduced into the final result for $\Delta\lambda$ by utilizing the simplified representation. $I_1(S_{A1} + S_{B1})/2 - I_2(S_{A2} + S_{B2})/2$ is measured at $t=0$, and $I_1(S_{A1} + S_{B1})/2$ is measured independently so that $I_2(S_{A2} + S_{B2})/2$, required for the determination of $\Delta\lambda$, is known.

¹⁴ R. T. Birge, Phys. Rev. 40, 225, 226 (1932). $c^{\frac{1}{2}}$ as used here is 1.06745 times $c^{\frac{1}{2}}$ in Birge's notation, since standard deviations, σ , rather than probable errors, are being evaluated.

The standard deviation¹⁵ of the difference current i is proportional to $(2N_0)^{\frac{1}{2}}$, where N_0 is the mean number of ionization events occurring from a single source during the duration D of a single determination of i . Hence, the deviation of $\Delta i = (i_{12} - i_{21})/2$ varies as $N_0^{\frac{1}{2}}$ and that of y_k , i.e., σ_k , is proportional to $(N_0 e^{\lambda t})^{\frac{1}{2}}$. The data were weighted with $p_k = c/\sigma_k^2 = c e^{-\lambda t_k}$ where $c^{\frac{1}{2}}$ is the standard deviation of y for $x=0$ (making $p_0=1$). The experimental data support the assumption implicit above that the main contribution to the final error in the determination of $\Delta\lambda/\lambda$ is the statistical fluctuation in the number of separate ionization events which have occurred during an observation.

With the quantities I_0 , λ , and b , one can now calculate the desired result,

$$\Delta\lambda/\lambda = b/I_0\lambda. \quad (5)$$

If y is measured for two values of x only, $x=0$ and $x=\omega$, the resulting error in b ,

$$\sigma_b = c^{\frac{1}{2}}(e^{\omega\lambda} + 1)^{\frac{1}{2}}/\omega, \quad (6)$$

has a minimum when $(\omega\lambda - 2) = 2e^{-\omega\lambda}$, i.e., for $\omega = 2.22/\lambda$. However, considerably more than two readings were desirable in the actual experiments. If y is measured at equidistant points $x=0, \tau, 2\tau, \dots, n\tau$, where the mean life $\tau = 1/\lambda$, the coefficient κ , defined by $\sigma_b = \kappa c^{\frac{1}{2}}/\tau$, decreases with increasing n as shown by the open circles in Fig. 1, whereas κ for only two measurements at $x=0$ and $x=\omega$ is shown in the solid curve. In some experimental runs on Tc^{99m} it was possible to go through all the permutations of the sources in less than the interval τ ; in others involving four or more sources more time was required.

In an actual experiment the duration D of a single measurement is $i \cong 1000$ seconds $\cong 0.05T_{\frac{1}{2}}$. This corresponds to a measurement of approximately 4×10^8 ionization events from a single source of strength $I = 10^4$ divisions ($\sim 100 \mu\text{C}$ Ra equivalent) so that $c^{\frac{1}{2}}$ is equal

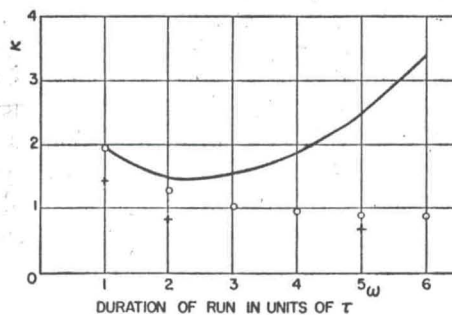


FIG. 1. The coefficient κ occurring in the expression for the standard deviation in b , $\sigma_b = \kappa c^{\frac{1}{2}}/\tau$. The quantity $c^{\frac{1}{2}}$ is the standard deviation of the measurement at $x=0$. The solid curve represents the case where measurements are taken only at $x=0$ and $x=\omega$. The small circles represent the cases where the measurement is made at all integral multiples of τ up to ω . The crosses represent the three cases of eleven equally spaced measurements terminating, respectively, at τ , 2τ , and 5τ .

¹⁵ E. V. Schweidler, First International Congress of Radiology and Ionization (Liège, 1905).